Sampling and Uncertainty

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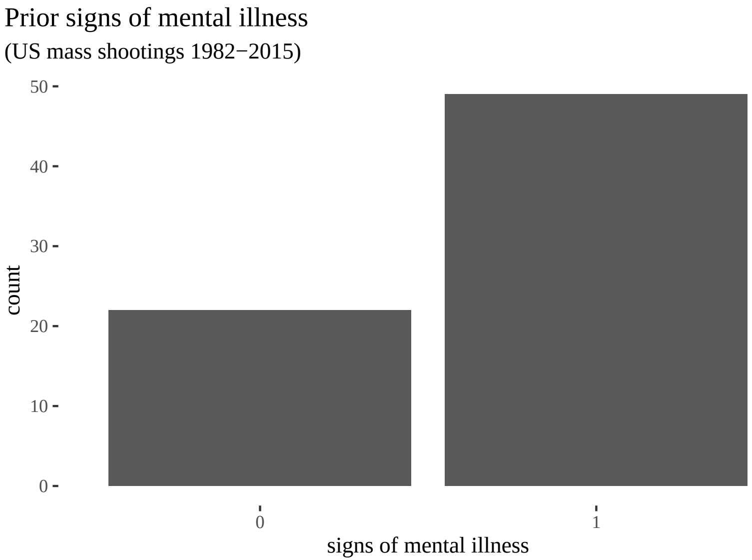
Proportion estimates and sampling

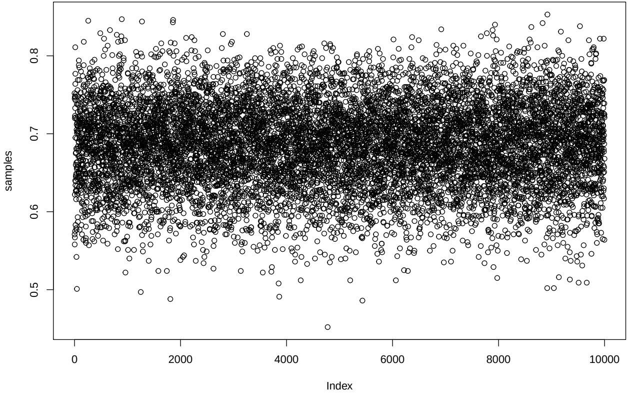


Proportion estimates and sampling

```
p_grid <- seq(0,1, length.out = 1001)
prior <- rep( 1, 1001)

likelihood <- dbinom( sum(sh$mental),
                      size = nrow(sh), prob = p_grid)

posteriorUnnormalized <- prior * likelihood

posterior <- posteriorUnnormalized / sum(posteriorUnnormalized)
```

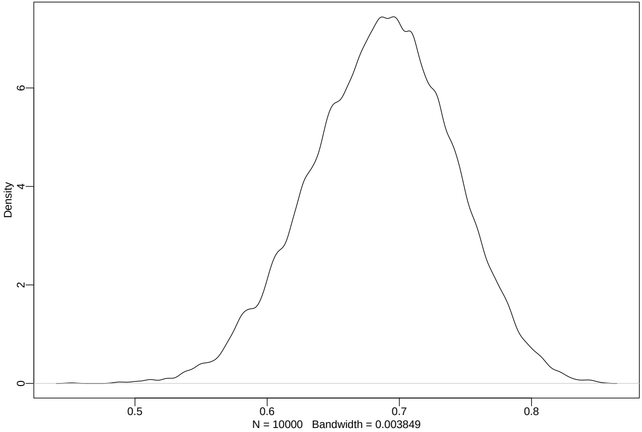
Proportion estimates and sampling

```
samples <- sample(p_grid, prob = posterior, size = 1e4,
                  replace = TRUE)
plot(samples)
```



Proportion estimates and sampling

```
dens(samples)
```



Proportion estimates and sampling

```
sum(posterior[p_grid > .6])
```

```
## [1] 0.9357886
```

```
sum(samples > .6) / 1e4
```

```
## [1] 0.9384
```

```
sum(samples > .6 & samples < .7) / 1e4
```

```
## [1] 0.5246
```

Proportion estimates and sampling

```
quantile( samples, c(.1,.9))
```

```
##   10%   90%
## 0.615 0.754
```

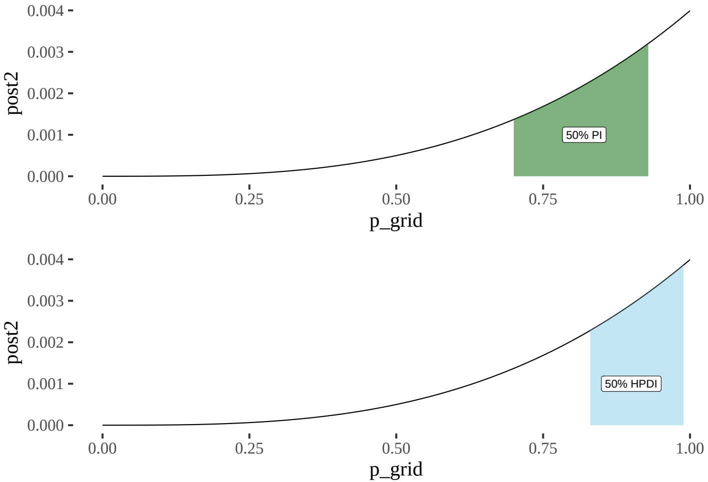
```
PI(samples, .8)
```

```
##   10%   90%
## 0.615 0.754
```

```
HPDI(samples, .8)
```

```
##  |0.8  0.8|
## 0.617 0.755
```

PI vs HPDI


0.004
0.003
0.002
0.001
0.000
post2
50% PI
0.00
0.25
0.50
0.75
1.00
p_grid
0.004
0.003
0.002
0.001
0.000
post2
50% HPDI
0.00
0.25
0.50
0.75
1.00
p_grid

Now with model building

```
sh$WEAPONSOBTAINEDLEGALLY
```

```
##  [1] "Yes" "Yes" "No"  ""    "Yes" "Yes" "Yes" "Yes" "Yes" "No"  "Yes" "Yes"
## [13] "Yes" "Yes" "Yes" "Yes" "Yes" "Yes" "Yes" "Yes" "No"  "Yes" "Yes" "Yes"
## [25] "Yes" "Yes" "No"  "No"  "Yes" "Yes" "No"  "Yes" "Yes" "Yes" "No"  "Yes"
## [37] "Yes" "Yes" "Yes" "Yes" "Yes" "Yes" "Yes" "Yes" "No"  "No"  "No"  "Yes"
## [49] "Yes" "No"  "Yes" "Yes" "Yes" ""    "Yes" "Yes" "No"  "Yes" "Yes" "Yes"
## [61] "Yes" "Yes" "Yes" "Yes" "Yes" "Yes" "Yes" "Yes" "Yes" "No"  "Yes"
```

Now with model building

```
legal <- sum(sh$WEAPONSOBTAINEDLEGALLY == "Yes")
illegal <- sum(sh$WEAPONSOBTAINEDLEGALLY == "No")
total <- legal + illegal

datweapons = list (legal = legal, illegal = illegal,
                   total = total)


weaponsModel <- ulam(
  alist(
    legal ~ dbinom( total , theta),
    theta ~ dunif(0,1)
  ) ,
  data= datweapons )
```

Now with model building

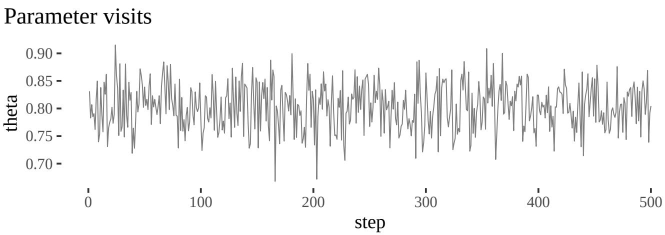
```r
precis(weaponsModel)
```

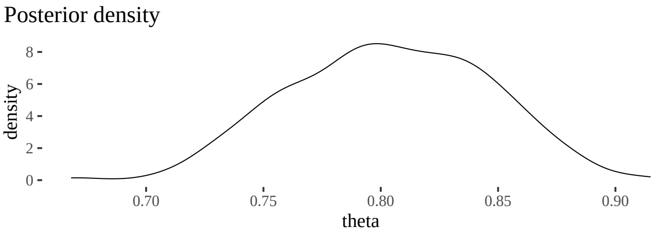
```
##              result
## mean     0.80361414
## sd       0.04195767
## 5.5%     0.73655486
## 94.5%    0.86878227
## n_eff  214.86647318
## Rhat     0.99801618
```

```r
weaponsSamples <- as.data.frame(extract.samples(weaponsModel))

weaponsSamples$step <- 1:500

head(weaponsSamples)
```

```
##      theta step
## 1 0.831363    1
## 2 0.782842    2
## 3 0.806978    3
## 4 0.785337    4
## 5 0.790650    5
## 6 0.762034    6
```

Now with model building





Beyond binomial: lots of small factors

```
set.seed(312)
runif(1,-1,1)
```

```
## [1] 0.5832919
```

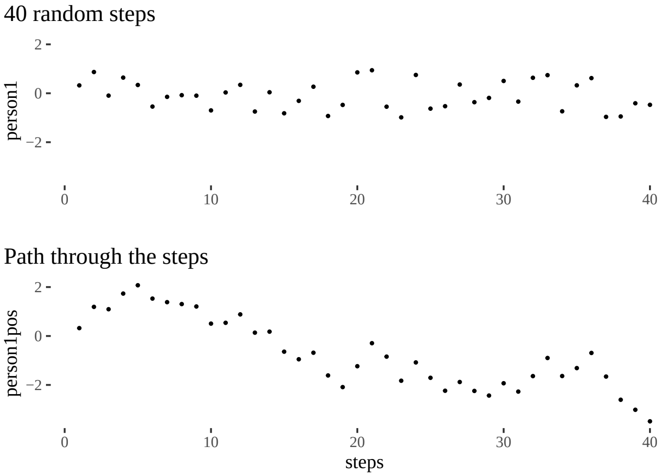
```
person1 <- runif(40,-1,1)
person1[1:15]
```

```
##  [1]  0.32220310  0.86845868 -0.09645973  0.63976928  0.33923684 -0.54382513
##  [7] -0.14640792 -0.07832506 -0.09818215 -0.70031680  0.03328715  0.34389619
## [13] -0.74667117  0.04139647 -0.81864988
```

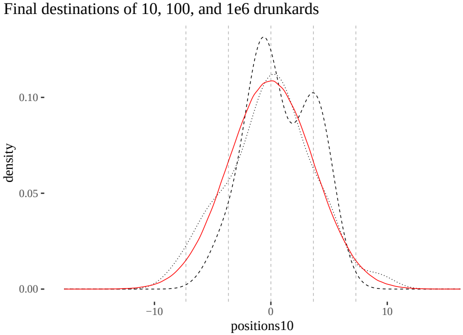
```
person1pos <- cumsum(person1)
person1pos
```

```
##  [1]  0.3222031  1.1906618  1.0942021  1.7339713  2.0732082  1.5293830
##  [7]  1.3829751  1.3046501  1.2064679  0.5061511  0.5394383  0.8833345
## [13]  0.1366633  0.1780598 -0.6405901 -0.9509429 -0.6835546 -1.6124981
## [19] -2.0885637 -1.2350482 -0.2942103 -0.8431846 -1.8282965 -1.0799415
## [25] -1.7083775 -2.2375286 -1.8804667 -2.2449167 -2.4347412 -1.9316288
## [31] -2.2723666 -1.6391910 -0.8989080 -1.6362535 -1.3121734 -0.6925716
## [37] -1.6578202 -2.6061456 -3.0153709 -3.4872044
```

Beyond binomial: lots of small factors



Beyond binomial: lots of small factors




```
sd(positions1e6)
```

```
## [1] 3.651485
```

```
mean(abs(positions1e6) < abs(sd(positions1e6)) )
```

```
## [1] 0.681449
```

```
mean(abs(positions1e6) < 2 * abs(sd(positions1e6)) )
```

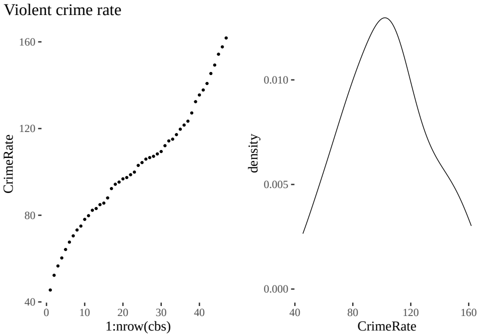
```
## [1] 0.954788
```

Crime rates and normal distribution

```
cbs <-  read.csv(file = "../../datasets/CrimeByState.csv")
#these are registered violent incidents per 100k citizens
cbs$CrimeRate
```

```
##  [1]  45.5  52.3  56.6  60.3  64.2  67.6  70.5  73.2  75.0  78.1  79.8  82.3
## [13]  83.1  84.9  85.6  88.0  92.3  94.3  95.3  96.8  97.4  98.7  99.9 103.0
## [25] 104.3 105.9 106.6 107.2 108.3 109.4 112.1 114.3 115.1 117.2 119.7 121.6
## [37] 123.4 127.2 132.4 135.5 137.8 140.8 145.4 149.3 154.3 157.7 161.8
```

```
cbsPlot <- grid.arrange(ggplot(cbs)+geom_point(aes(x=1:nrow(cbs),y = CrimeRate))+th+
            ggtitle("Violent crime rate"),
ggplot(cbs)+geom_density(aes(x=CrimeRate))+th, ncol=2)
```



Crime rates and normal distribution

```
dat <- list(
  rate = cbs$CrimeRate
)

set.seed(123)
meanModel <- quap(
  alist(
  rate ~ dnorm( mu , sigma ) ,
  mu ~ dnorm( 100 , 100 ) ,
  sigma ~ dunif( 0 , 50 )
  ), data = dat
)
```

Crime rates and normal distribution

```
precis(meanModel)
```

```
##             mean       sd      5.5%     94.5%
## mu    102.79877 4.165765 96.14107 109.45646
## sigma  28.58386 2.948141 23.87216  33.29556
```

```
pred <- sim(meanModel)

str(pred)
```

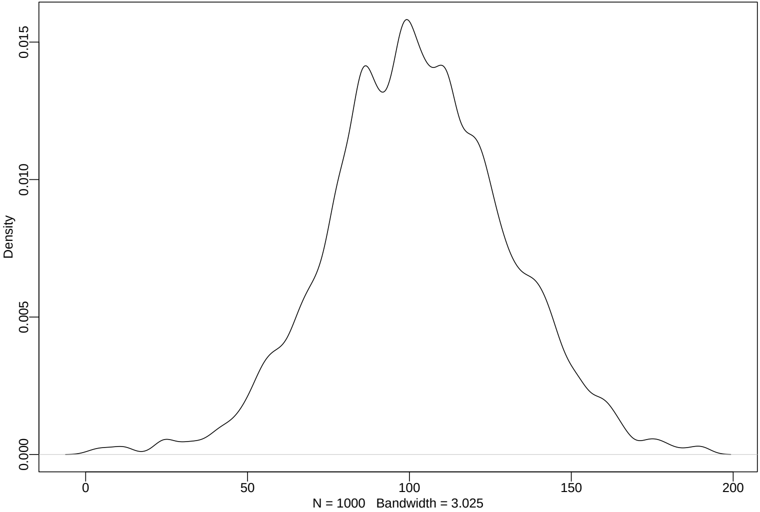
```
##  num [1:1000, 1:47] 123 160 110 112 124 ...
```

```
pred[1:5, 1:5]
```

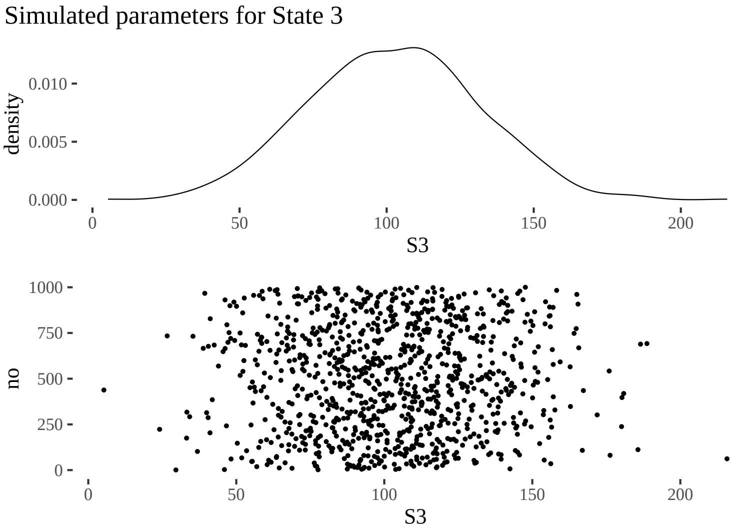
```
##          [,1]      [,2]      [,3]      [,4]      [,5]
## [1,] 122.5506  67.21098  29.59797 150.41422 158.88619
## [2,] 159.9094 115.09314  77.61558 193.44488  99.14939
## [3,] 110.2740 122.49128  45.99694 136.25065 106.91446
## [4,] 111.9459 112.71564 103.83174  92.10859  76.17974
## [5,] 123.6168  92.76183  92.35412 106.23952  95.85145
```

Crime rates and normal distribution

```
dens (pred[,1])
```



Crime rates and normal distribution



Crime rates and normal distribution

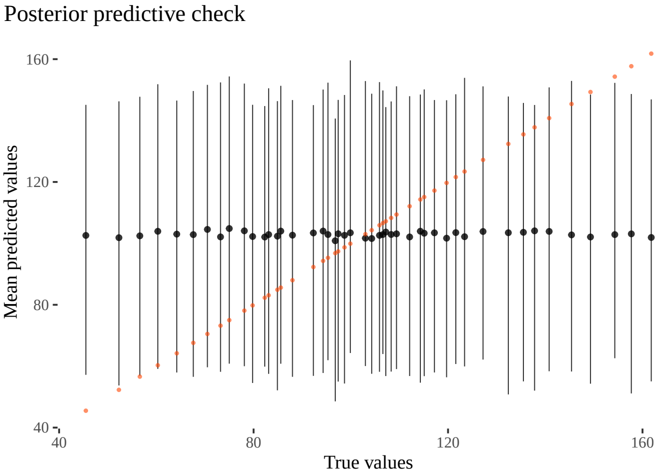
```r
(meanpreds <- apply(pred, MARGIN = 2, FUN = mean))
```

```
##  [1] 102.5905 101.8595 102.4534 103.9375 103.0147 102.8535 104.5446 102.1100
##  [9] 104.8375 104.0880 102.2362 102.0704 102.8730 102.3484 103.9993 102.6737
## [17] 103.4009 104.0101 102.8792 100.8548 103.1379 102.6278 103.4506 101.6893
## [25] 101.5838 102.6099 102.8824 103.6936 102.8997 103.1187 102.0964 103.9570
## [33] 103.3063 103.4443 101.6891 103.5112 102.1769 103.8713 103.4748 103.6042
## [41] 104.0845 103.9004 102.7667 102.0846 102.8636 103.0971 101.9225
```

```r
hpdipreds <- as.data.frame(t(apply(pred, MARGIN = 2, FUN = HPDI)))
head(hpdipreds, n=10)
```

```
##       |0.89    0.89|
## 1  57.18970 145.1188
## 2  53.71864 146.2659
## 3  56.92366 147.7539
## 4  59.12198 151.8405
## 5  57.94922 146.5091
## 6  56.53628 149.6273
## 7  59.64962 151.6313
## 8  58.17247 152.4303
## 9  60.82817 154.3709
## 10 59.99766 152.0494
```

Crime rates and normal distribution



Levels of uncertainty

```
rate ~ dnorm( mu , sigma ) ,
mu ~ dnorm( 100 , 100 ) ,
sigma ~ dunif( 0 , 50 )
```

```
##            mean       sd     5.5%     94.5%
## mu    102.79877 4.165765 96.14107 109.45646
## sigma  28.58386 2.948141 23.87216  33.29556
```

Levels of uncertainty

```r
est <- extract.samples( meanModel )
pred <- sim( meanModel)

head(est)
```

```
##          mu    sigma
## 1 103.88316 21.73971
## 2  95.28742 31.64544
## 3  92.49712 31.12295
## 4 100.35946 26.08824
## 5  97.17260 31.52742
## 6 107.50812 29.07011
```

```r
str(pred)
```

```
##  num [1:1000, 1:47] 122.3 159.6 126.4 77.5 69.8 ...
```

Levels of uncertainty